Antwoord model tentamen "Lie groups in physics" 4-11-13

punten

1a) 50% general form of elements in $O(1,1)$: $O = \begin{pmatrix} a & b \\ bD & aD \end{pmatrix}$ with $a^2-b^2=1$, $a,b\in\mathbb{R}$ and $D=\pm1$

50% group: $(O_1O_2)^T = g(O_1O_2)^{-1}g^{-1}$

Non Abelian: $O_1O_2 \neq O_2O_1$ if $D_2\neq D_1$ and $D_2 \neq 1$

noncompact: $1\le a \le \infty$ or $-\infty \le a \le -1$ $\Rightarrow$ unbounded parameter set

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1b) 50% connected components: $H_+^\uparrow$: $a\geq 1$, $D=1$ (connected subgroup)

$H_-^\uparrow$: $a\geq1$, $D=-1$

$H_+^\downarrow$: $a\leq -1$, $D=1$

$H_-^\downarrow$: $a\leq -1$, $D=-1$.

50%

$H_-^\uparrow = \begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix} H_+^\uparrow$

$H_+^\downarrow = \begin{pmatrix} -1 & 0 \\ 0 & -1\end{pmatrix} H_+^\uparrow$

$H_-^\downarrow = \begin{pmatrix} -1 & 0 \\ 0 & 1\end{pmatrix} H_+^\uparrow$

hence cosets of the connected subgroup.

Factor group $= \mathbb{Z}_2 \oplus \mathbb{Z}_2 \cong D_2$

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1c) Use Schur's lemma to conclude it is indeed an irrep because only matrix B that commutes with all O is $\alpha \mathbb{1}$ $\alpha\in\mathbb{R}$.

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1d) 50% Lie algebra L obtained from infinitesimal elements around $\mathbb{1}$:

$$\begin{pmatrix} \sqrt{1-b^2} & b \\ b & \sqrt{1-b^2}\end{pmatrix}$$ for small b.

$\frac{\partial}{\partial b}$ at $b=0$ yields $\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}$, the generator of this 1 dimensional Lie group.

50% One can use Schur's lemma to show it is not a irrep of the Lie algebra.

Another way is: connected subgroup is Abelian, hence all irreps of $H_+^\uparrow$ and hence of L are 1-D.

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alternatief: $O=e^A$ such that $A^T = -gAg$ $\rightarrow A = b\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}$

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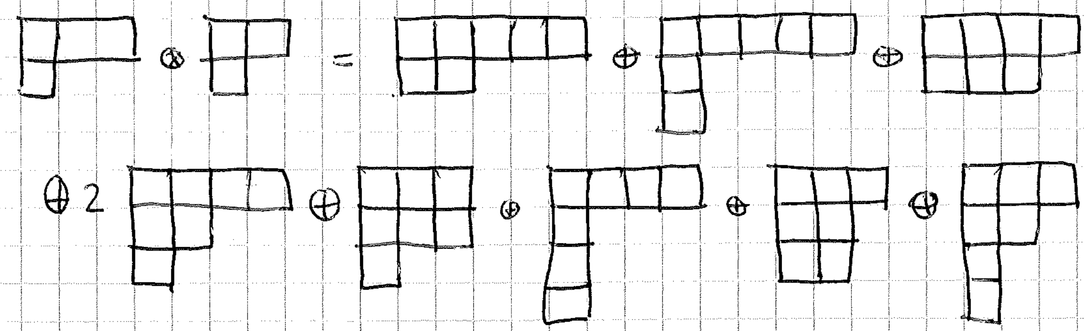
2a)



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2b) $su(2)$: $3 \otimes 2 = 4 \oplus 2$ — 33%

$su(3)$: $15_1 \otimes 8 = 42 \oplus 15_2 \oplus 24 \oplus 2 \cdot 15_1 \oplus 6 \oplus 3$ — 67%

because has added issue of complex conj rep or not: $15_2 \neq 15_1^*$

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2c) $3 \otimes 2 = 4 \oplus 2$ corresponds to $j_1 = 1$ & $j_2 = \frac{1}{2}$ added to give $j_3 = \frac{3}{2}$ or $j_3 = \frac{1}{2}$

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3a) $$\underbrace{(M^{01})^\alpha{}_\beta = \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = k^1}_{50\%} \qquad \underbrace{(M^{23})^\alpha{}_\beta = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} = J^1}_{50\%}$$

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3b) $$e^{-i\chi M^{01}} = \begin{pmatrix} \cosh\chi & \sinh\chi & 0 & 0 \\ \sinh\chi & \cosh\chi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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90 geeft : $\frac{90}{10} + 1 = 10$